Quadrature for self-similar distributions on R^d

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We study numerical integration of q-times differentiable functions on $\mathbb{R}^d$ for a probability measure that is self-similar with respect to m affine contraction mappings $S_1, \ldots, S_m \colon \mathbb{R}^d \to \mathbb{R}^d$ and corresponding probability weights $\rho_1, \ldots, \rho_m$. Under mild conditions on the contractions we provide lower bounds for the worst case errors of deterministic as well as randomized algorithms in terms of the worst case (average) number of function evaluations that are used. The matching upper bounds are obtained by composite quadrature rules, which are easy to implement and are based on divide and conquer strategies that are adapted to the structure of the self-similarity. The optimal order of convergence is characterized in terms of the similarity dimension of the contractions $\rho_1 S_1^q, \ldots, \rho_m S_m^q$. This is joint work with Steffen Dereich (University of Münster).